PRODUCTION AND OPERATIONS MANAGEMENT
2023/2024

Lisbon School of Economics & Management

UNIVERSIDADE DE LISBOA

QM_Exer. 1

The SWEETCAKE company produces three different cake rolls, each of a different flavour (vanilla, strawberry and chocolate). Each flavour is produced on a different production line, that is, chocolate on line 1, strawberry on line 2 and vanilla on line 3. After an analysis of the productive process the Quality Director of the company decides that one of characteristics that should be controlled, through control chats for variables, is the weight of the cakes. With this purpose, he collected 26 samples of 5 chocolate cake rolls each from line 1 and registered the average weight and the range for each sample:

Sample	1	2	3	4	5	6	7	8	9	10	11	12	13
Average Weight	301	307	308	302	311	310	301	310	305	305	310	303	310
Range	10	15	8	7	21	9	12	6	17	11	18	9	15
Sample	14	15	16	17	18	19	20	21	22	23	24	25	26
Average Weight	310	305	312	304	310	306	312	302	307	310	303	311	308
Range	14	18	10	12	15	8	16	15	19	10	15	13	18

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Sample	1	2	3	4	5	6	7	8	9	10	11	12	13
Average Weight	301	307	308	302	311	310	301	310	305	305	310	303	310
Range	10	15	8	7	21	9	12	6	17	11	18	9	15
Sample	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**
Average Weight	310	305	312	304	310	306	312	302	307	310	303	311	308
Range	14	18	10	12	15	8	16	15	19	10	15	13	18

$UCL_X = \bar{\bar{X}} + A_2\bar{R}$ $LCL_X = \bar{\bar{X}} - A_2\bar{R}$ $UCL_R = D_4\bar{R}$ $UCL_R = D_3\bar{R}$

Sample size: n = 5 ➔ $A_2 = 0.577$; $D_3 = 0$ and $D_4 = 2.115$.

Number of samples: k = 26 ➔ $\bar{X} = \frac{\sum_{i=1}^{26} \bar{X}_i}{26} = 307.04$ $\bar{R} = \frac{\sum_{i=1}^{26} R_i}{26} = 13.12$

a) Determine the control limits for the $\bar{\text{X}}$-chart and R-chart.

QM_Exer. 1

Sample	1	2	3	4	5	6	7	8	9	10	11	12	13
Average Weight	301	307	308	302	311	310	301	310	305	305	310	303	310
Range	10	15	8	7	21	9	12	6	17	11	18	9	15
Sample	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**
Average Weight	310	305	312	304	310	306	312	302	307	310	303	311	308
Range	14	18	10	12	15	8	16	15	19	10	15	13	18

$UCL_X = \bar{\bar{X}} + A_2\bar{R}$ $LCL_X = \bar{\bar{X}} - A_2\bar{R}$ $UCL_R = D_4\bar{R}$ $UCL_R = D_3\bar{R}$

The control limits for the **X-chart** are:

$$UCL_{\bar{X}} = \bar{X} + A_2 \times \bar{R} = 307.04 + 0.577 \times 13.12 = 314.61$$

$$LCL_{\bar{X}} = \bar{X} - A_2 \times \bar{R} = 307.04 - 0.577 \times 13.12 = 299.47$$

The control limits for the **R-chart** are:

$$UCL_R = D_4 \times \bar{R} = 2.115 \times 13.12 = 27.74$$

$$LCL_R = D_3 \times \bar{R} = 0.0 \times 13.12 = 0.0$$

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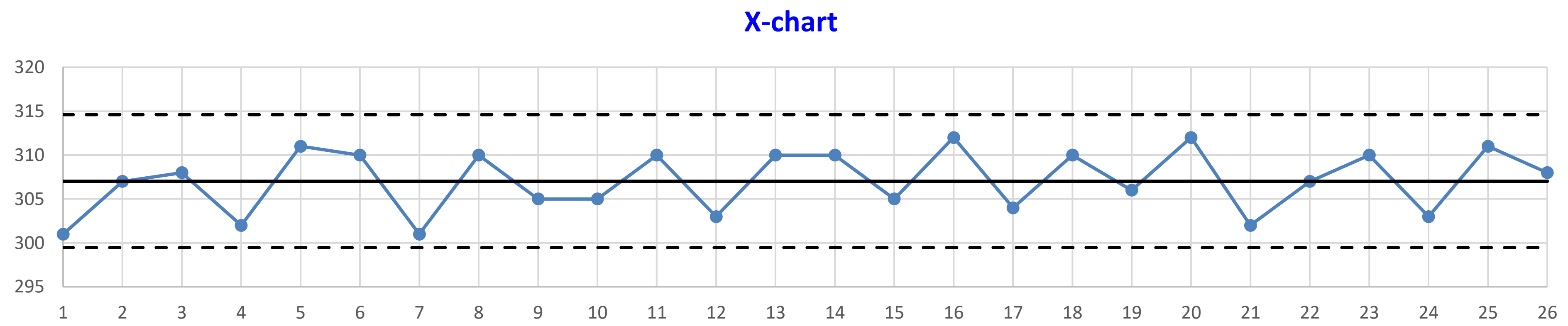
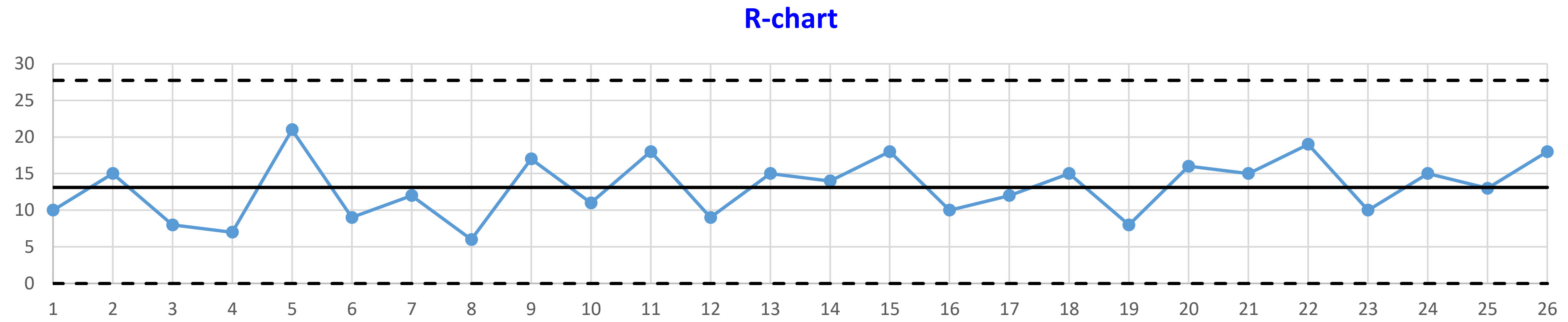




Since there is no sign of out of control samples on both charts, one can say that the calculated limits are the appropriate limits to control the process, and therefore **the process is in statistical control.**

QM_Exer. 1

b) In your opinion, do you think that the company should use the same control limits determined in a) to control the weight of the vanilla and strawberry cakes? Justify your answer.

No. The control limits are characteristic of each process. The same limits should only be maintained if the mean and standard deviation of the process for vanilla and strawberry cakes were the same as for chocolate cakes.

QM_Exer. 1

c) According to the customers' requirements, the company established that the upper and lower specification limits for the cakes' weight is 285 grams and 315 grams, respectively. Do you think that the process is capable of producing chocolate cake rolls according to the established specifications? Justify your answer presenting all the assumptions made.

The mean and standard deviation of the process, estimated from the control charts are ($n = 5$; $d_2 = 2.326$):

$$\mu = \bar{\bar{X}} = 307.04 \text{ and } \hat{\sigma} = \frac{\bar{R}}{d_2} = \frac{13{,}12}{2.326} = 5.64\ grams$$

To assess the Process Capability we need to calculate the C_{pk}:

$$C_{pk} = \min(C_{pki}; C_{pks})$$

QM_Exer. 1

$$C_{pk} = \min(C_{pki}; C_{pks})$$

$$\boldsymbol{C_{pki}} = \frac{\mu - LSL}{3\hat{\sigma}} = \frac{307.04 - 285}{3 \times 5.64} = \mathbf{1.3} \quad \boldsymbol{C_{pks}} = \frac{USL - \mu}{3\hat{\sigma}} = \frac{315 - 307.04}{3 \times 5.64} = \mathbf{0.47}$$

$$\boldsymbol{C_{pk}} = \min(C_{pki}; C_{pks}) = (1.3; 0.47) = \mathbf{0.47}$$

As C_{pk} is less than 1, the process is not capable of producing according to specifications. Moreover, we can see that the process is not centered ➔ ($C_{pki} \neq C_{pks}$).

$$C_p = \frac{USL - LSL}{6 \times \sigma} = \frac{315 - 285}{6 \times 5.64} = 0.89$$

The process is not centered ($C_{pki} \neq C_{pks}$), and the variability is greater than the tolerance $C_p < 1$.

End of class 2021-02-17.

QM_Exer. 2

TEXTILUNIFORM is a manufacturing company that produces high quality School Uniforms T-Shirts. Daily a sample of 100 T-Shirt is carefully inspected and the number of defective T-shirts is recorded.

The following table shows a sequence of 26 results recorded over time (26 days).

Day	1	2	3	4	5	6	7	8	9	10	11	12	13
Number of defective T-shirts	8	8	6	8	9	5	7	2	3	10	5	7	4
Day	14	15	16	17	18	19	20	21	22	23	24	25	26
Number of defective T-shirts	15	6	8	2	1	7	9	11	9	6	5	8	3

QM_Exer. 2

a) According to the data recorded, which is the appropriate control chart? Determine the 3-sigma control limits for the suggested control chart.

Number of defective T-shirts ➔ the appropriate control chart is the ***p*-chart** (attribute chart), we are in a pass/fail situation.

Sample size: n = 100; Number of samples: k = 26.

Day	1	2	3	4	5	6	7	8	9	10	11	12	13
Number of defective T-shirts	8	8	6	8	9	5	7	2	3	10	5	7	4
p_i	0.08	0.08	0.06	0.08	0.09	0.05	0.07	0.02	0.03	0.10	0.05	0.07	0.04
Day	14	15	16	17	18	19	20	21	22	23	24	25	26
Number of defective T-shirts	15	6	8	2	1	7	9	11	9	6	5	8	3
p_i	0.15	0.06	0.08	0.02	0.01	0.07	0.09	0.11	0.09	0.06	0.05	0.08	0.03

The average proportion of defective T-shirts is given by:

$$\bar{p} = \frac{\sum_{i=1}^{26} p_i}{26} = \frac{1.72}{26} = 0.066$$

QM_Exer. 2

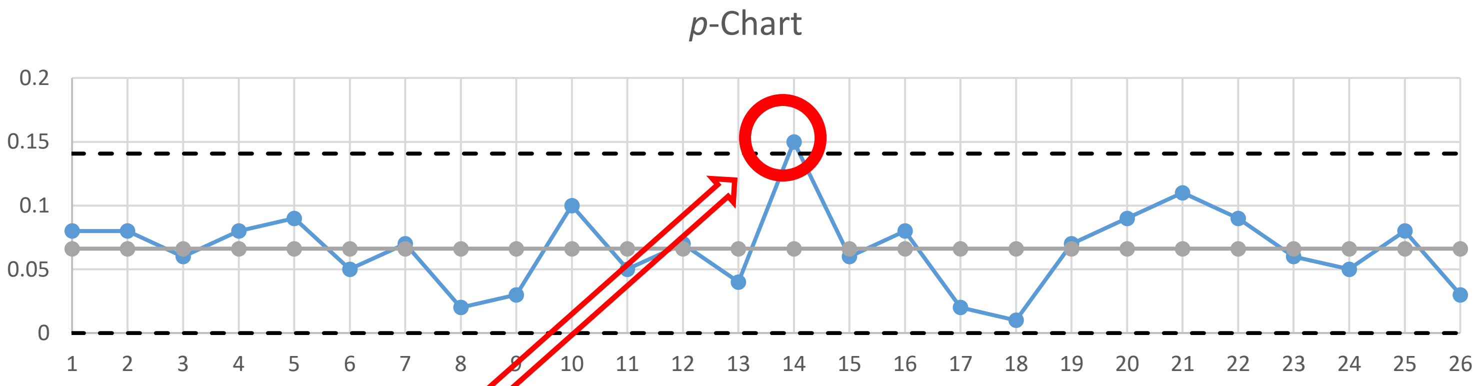
The control limits and the center line for the ***p*-chart** are given by:

$$\mathbf{UCL_p} = \overline{p} + 3 \times \sqrt{\frac{\overline{p}(1-\overline{p})}{n}} = 0.066 + 3 \times \sqrt{\frac{0.066 \times (1-0.066)}{100}} = \mathbf{0.14}$$

$$\mathrm{CL_p} = \overline{p} = 0.066$$

$$\mathbf{LCL_p} = \overline{p} - 3 \times \sqrt{\frac{\overline{p}(1-\overline{p})}{n}} = 0.066 - 3 \times \sqrt{\frac{0.066 \times (1-0.066)}{100}} = \mathbf{0.0}$$

It is considered zero since there are no negative percentages of defective.



Since the Sample 14 goes outside the control limits (special cause of variation), **these limits cannot be used to control the process.**

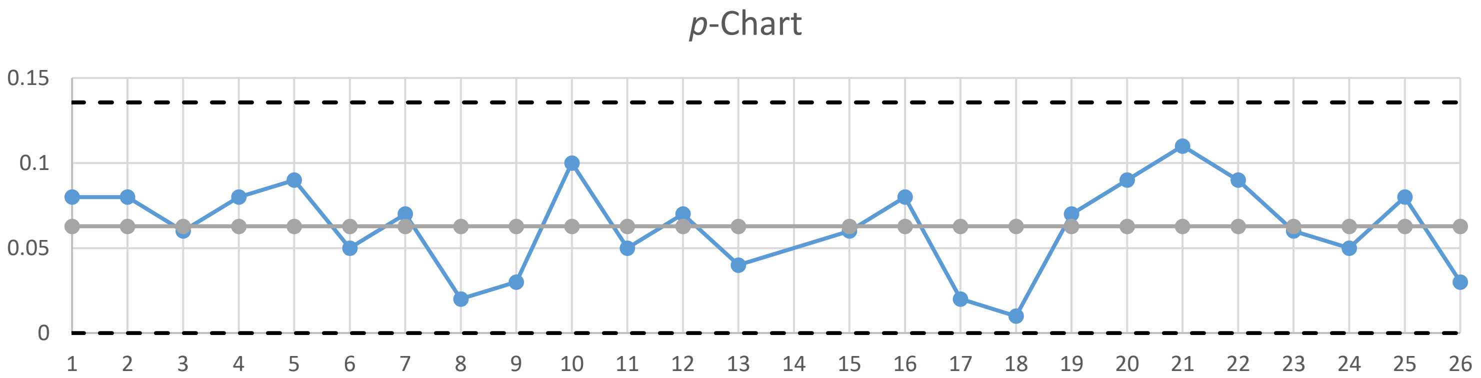
QM_Exer. 2

The new control limits for the *p*-chart, obtained after removing Sample 14, are as follows:

$$\mathbf{UCL_p} = \bar{p} + 3 \times \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.063 + 3 \times \sqrt{\frac{0.063\times(1-0.063)}{100}} = \mathbf{0.136}$$

$$\mathrm{CL_p} = \bar{p} = 0.063 \longrightarrow \bar{p} = \frac{(1.72 - 0.15)}{25} = 0.063$$

$$\mathbf{LCL_p} = \bar{p} - 3 \times \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.063 - 3 \times \sqrt{\frac{0.063\times(1-0.063)}{100}} = \mathbf{0.0}$$



Now, only **common causes of variation** are present. Thus, **these limits are appropriate to control the process.**

QM_Exer. 2

b) Suppose that the number of defective T-shirts on the next four days are 10, 6, 5 and 16. Is the process in control? Identify the causes of variation present in the process.

Day	27	28	29	30
p_i	0.10	0.06	0.05	0.16

Since Sample 30 goes outside the upper control limit (0.16 > UCL), there are **special causes of variation** that must be analyzed.

The process is not in statistical control.

QM_Exer. 3

CORKSTOP produces thousands of cork stoppers per day. The quality manager of CORKSTOP decided to have a closer control on the production process to discover the cause of the defects on the cork stoppers. Therefore, he started to collect 20 samples of 100 cork stoppers each. The number of defects on a given day is as follows:

Sample	1	2	3	4	5	6	7	8	9	10
# of defects	6	4	5	1	4	2	5	3	3	2

Sample	11	12	13	14	15	16	17	18	19	20
# of defects	6	1	8	5	7	4	3	7	2	4

a) What control chart do you find most appropriate to analyze the collected data?

Since we intend to control the number of defects, the most appropriate control chart is ***c*-Chart.**

QM_Exer. 3

b) Draw the appropriate control chart(s) so that the control limits include 99.73% of the random variation on the entire process (3 sigma limits).

Sample size: n = 100; Number of samples: k = 20.

$$\bar{c} = \frac{\sum_{i=1}^{20} c_i}{20} = \frac{82}{20} = \mathbf{4.1}$$

$$\mathbf{UCL_c} = \bar{c} + 3 \times \sqrt{\bar{c}} = 4.1 + 3 \times \sqrt{4.1} = \mathbf{10.17}$$

$$\mathrm{CL_c} = 4.1$$

$$\mathbf{LCL_c} = \bar{c} - 3 \times \sqrt{\bar{c}} = 4.1 + 3 \times \sqrt{4.1} = \mathbf{0.0}$$

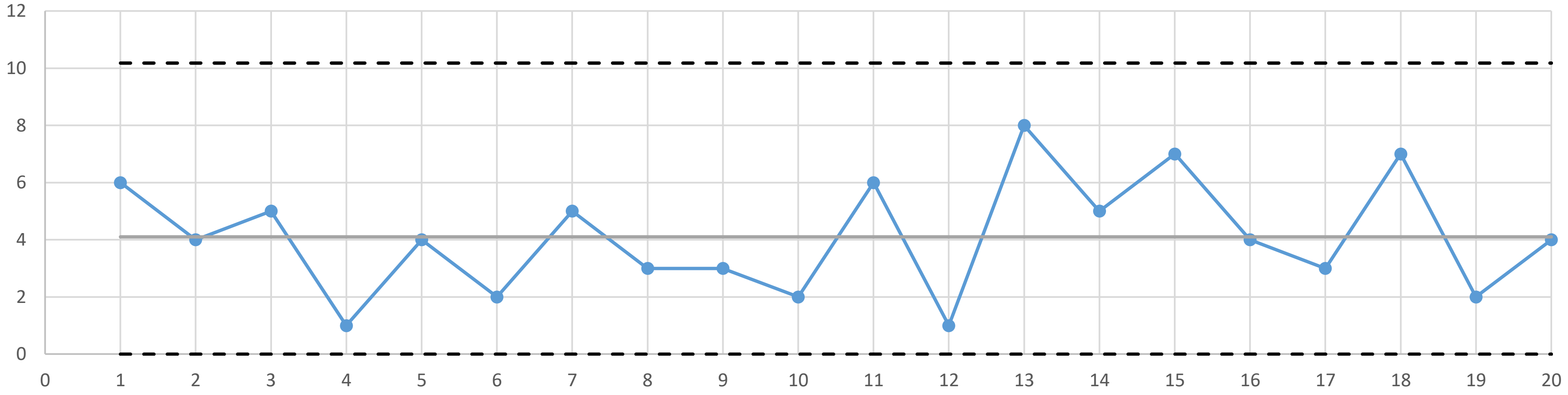
It is considered equal to zero because there are no negative defects.

QM_Exer. 3

$$\mathbf{UCL_c} = \bar{c} + 3 \times \sqrt{\bar{c}} = 4.1 + 3 \times \sqrt{4.1} = \mathbf{10.17}$$

$$\mathbf{LCL_c} = \bar{c} - 3 \times \sqrt{\bar{c}} = 4.1 + 3 \times \sqrt{4.1} = \mathbf{0.0}$$

***c*-Chart**



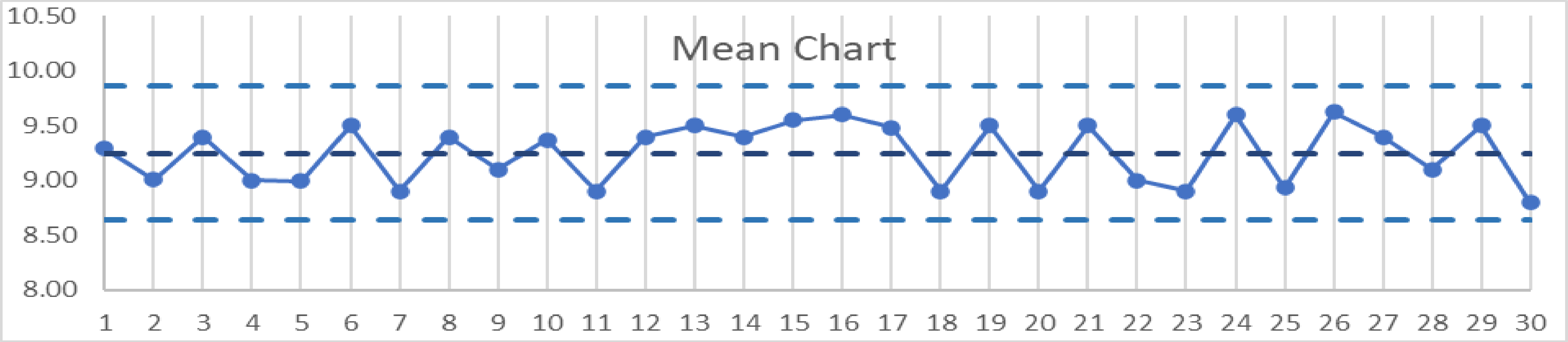
Only **common causes of variation** are present. Thus, **these limits are appropriate to control the process.**

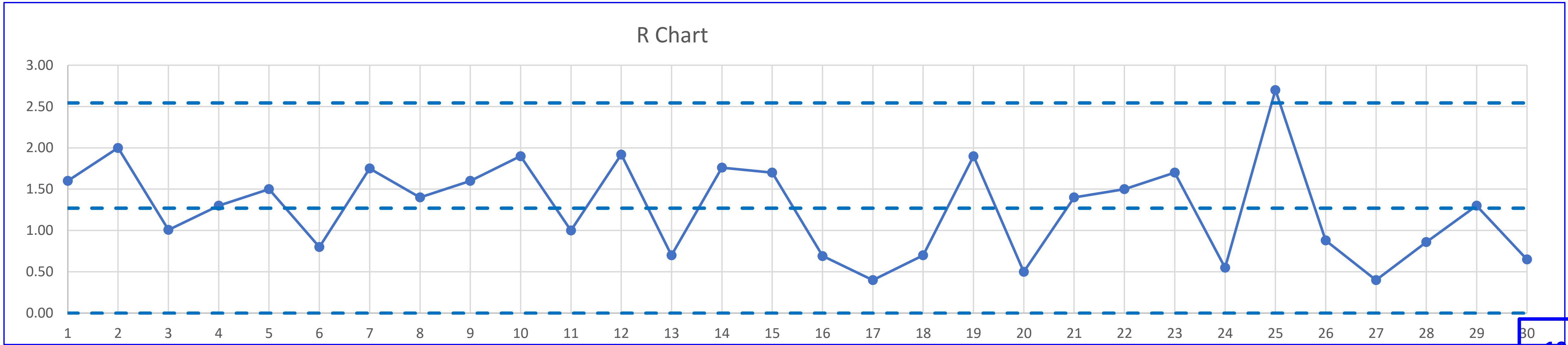
QM_Exer. 4

ELEChip company produces several types of chips for the electronic industry. After an analysis of the productive process the quality director of the company decides that one of characteristics that should be controlled, through control charts for variables, is the electrical resistance. With this purpose, he collected 30 samples of 6 chips each and registered the average and the range of the electrical resistance (in ohms) for each sample. Using this data, he calculated the control limits for the mean and range control chart (shown in the table below) and drew the respective graphs.

	Mean chart	Range chart
Central line	$CL = \overline{\overline{X}} = 9.25$ ohms	CL = 1.27 ohms
3 sigma limits	UCL= 9.86 ohms LCL = 8.64 ohms	UCL = 2.54 ohms LCL = 0 ohms

QM_Exer. 4


Mean Chart
10.50
10.00
9.50
9.00
8.50
8.00
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30


R Chart
3.00
2.50
2.00
1.50
1.00
0.50
0.00
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

QM_Exer. 4

a) In your opinion the control limits obtained by the quality manager (represented in the above graphs) are appropriate to start controlling the electrical resistance of the chips? Justify your answer. (Note: Is it not necessary to perform any calculation).

The control limits calculated by the quality director **are not appropriate to start controlling the characteristic "electrical resistance"**, since they were not calculated based only on samples that reflect common causes of variation.

By observing the Mean chart and the Range chart, one can see that there are signs of an out-of-control process, namely, 6 consecutive points (samples 12 to 17) above the central line in the Mean chart, and one point above the UCL in the Range chart (sample 25). **These signs show the presence of special causes of variation.**

QM_Exer. 4

b) Assume that the production process of the chips is under statistical control and that the characteristic electrical resistance follows a Normal distribution. The upper and lower specification limits for the electrical resistance are 12 ohms and 8 ohms, respectively. In your opinion, is the process able to produce according to the specifications? Justify your answer (Note: If you cannot estimate the mean and standard deviation of the process, assume that they are equal to 9.1 ohms and 0.6 ohms, respectively).

Assuming that the production process of the chips is under statistical control, and that the characteristic electrical resistance follows a Normal distribution, one can estimate the Mean and Standard Deviation as follows:

$\hat{\mu} = \bar{\bar{X}} = 9.25 \; and \; \hat{\sigma} = \frac{\bar{R}}{d_2} = \frac{1,27}{2.534} = 0.501 \; ohms$, with $d_2 = 2.534$ for a sample size of 6 (n=6).

The specification limits are USL=12 ohms and LSL=8 ohms. To evaluate the Process Capability, it is necessary to calculate the Cpk:

$$C_{pki} = \frac{\hat{\mu} - LSL}{3\hat{\sigma}} = \frac{9.25 - 8}{3 \times 0.501} = 0.832 \; and \; C_{pks} = \frac{USL - \hat{\mu}}{3\hat{\sigma}} = \frac{12 - 9.25}{3 \times 0.501} = 1.830$$

Therefore,

$$\boldsymbol{C_{pk}} = \min(C_{pki}; C_{pks}) = (0.832; 1.830) = \mathbf{0.832}, \text{ and } \boldsymbol{C_p} = \frac{USL - LSL}{6 \times \sigma} = \frac{12 - 8}{6 \times 0.501} = \mathbf{1.331}.$$

As Cpk is less than 1, the process is not able to produce according to the specifications. Although the process variability is lower than the tolerance (Cp > 1.0), it is not centered (Cpki ≠ CpkS).

QM_MChoice 1

1. The Quality Manager of an organic yogurt processing plant established, concerning the weight of yogurts, an upper specification limit of 500 grams and a lower specification limit of 420 grams. The production process of organic yogurt has a mean of 480 grams and a standard deviation of 12 grams. Assume that the weight of organic yogurt follows a normal distribution and that the process is under statistical control. Which of the following options is correct?

1		The process is centered, but it is not capable
2		The process is not centered, but it is capable
3		The process is not centered, and it is not capable
4		The process is centered, and capable

$$\boldsymbol{C_{pki}} = \frac{\mu - LSL}{3\hat{\sigma}} = \frac{480 - 420}{3 \times 12} = \mathbf{1.67} \qquad \boldsymbol{C_{pks}} = \frac{USL - \mu}{3\hat{\sigma}} = \frac{500 - 480}{3 \times 12} = \mathbf{0.56}$$

$$\boldsymbol{C_{pk}} = \min\left(C_{pki}; C_{pks}\right) = (1.67; 0.56) = \mathbf{0.56}$$

$$\boldsymbol{C_p} = \frac{USL - LSL}{6 \times \sigma} = \frac{500 - 420}{6 \times 12} = \mathbf{1.11}$$

3 – The process is not centered ($C_{pki} \neq C_{pks}$) and it is not capable.

QM_MChoice 1

1. The Quality Manager of an organic yogurt processing plant established, concerning the weight of yogurts, an upper specification limit of 500 grams and a lower specification limit of 420 grams. The production process of organic yogurt has a mean of 480 grams and a standard deviation of 12 grams. Assume that the weight of organic yogurt follows a normal distribution and that the process is under statistical control. Which of the following options is correct?

1		The process is centered, but it is not capable
2		The process is not centered, but it is capable
3	X	**The process is not centered, and it is not capable**
4		The process is centered, and capable

QM_MChoice 2

One of SATAR's competitive priorities is to reduce the delays of its flights. The company's quality director decided to control the number of delayed flights using control charts. With this purpose, he collected the following data:

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Delayed flights	4	8	12	8	2	8	18	11	22	4	6	8	4	4	13
Day	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Delayed flights	4	6	14	6	4	6	14	8	6	4	10	3	11	6	8

Which control chart do you find most appropriate to analyze the data collected by the quality manager?

1		Chart R
2		Chart p
3		Chart c
4		Chart X-bar

QM_MChoice 2

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Delayed flights	4	8	12	8	2	8	18	11	22	4	6	8	4	4	13
Day	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Delayed flights	4	6	14	6	4	6	14	8	6	4	10	3	11	6	8

Which control chart do you find most appropriate to analyze the data collected by the quality manager?

1		Chart R
2	X	Chart p
3		Chart c
4		Chart X-bar

QM_MChoice 2

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Delayed flights	4	8	12	8	2	8	18	11	22	4	6	8	4	4	13
Day	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Delayed flights	4	6	14	6	4	6	14	8	6	4	10	3	11	6	8

Which control chart do you find most appropriate to analyze the data collected by the quality manager?

1		Chart R
2	X	**Chart p**
3		Chart c
4		Chart X-bar

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